QUANTUM CORRECTIONS AND EXTREMAL BLACK HOLES

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Abstract

We consider static solutions of two dimensional dilaton gravity models as toy laboratories to address the question of the final fate of black holes. A nonperturbative correction to the CGHS potential term is shown to lead classically to an extremal black hole geometry, thus providing a plausible solution to Hawking evaporation paradox. However, the full quantum theory does not admit an extremal solution.

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1. The sound conjecture that quantum effects are able to cure the diseases of classical general relativity, such as singularities and the black hole evaporation paradox, has been recently disproven in certain cosmological [1,2], black hole [3,4] and colliding-wave [5] solutions to two dimensional dilaton gravity models coupled to conformal matter fields. On the contrary, according to [1,2], when the classical matter content is below a given threshold, rather the opposite may happen. Namely, certain nonsingular classical cosmological solutions develop quantum singularities in the weak coupling region, which suggests that they will not be removed in the full quantum theory. Moreover, the threshold and the leading behavior of the scale factor near the singularity are independent of the couplings. Similarly, classical solutions describing gravitational scattering of matter wave-packets develop curvature singularities vanishing only in the limit $\hbar \to 0$ [5].

In the black hole context, the original model proposed by Callan, Giddings, Harvey and Strominger (CGHS) [6] discusses Hawking radiation semiclassically by adding the conformal anomaly term, but cannot be solved exactly. The full quantization of the model was addressed by a number of authors [3,4,7]. The method followed restricts the effective action to a $c = 26$ conformal field theory that reduces to the semiclassical CGHS model in the weak coupling limit. It turns out to be possible to solve exactly the equations of motion of this conformal invariant theory. However, the model leads to unending thermal radiation and, consequently negative Bondi mass. This problem is related to the fact that the rate of Hawking radiation in these models does not change as the energy of the black hole is depleted. So, again, quantum effects seem not useful.

Two different proposals have been advanced in order to overcome this problem. The first one, originally suggested by Russo, Susskind and Thorlacius (RST) [8] consists of a modification of the CGHS effective action which admits analytic solutions. The modified model is again a conformal field theory, but now one of the fields has a restricted range of values $X \geq X_{cr}$. RST proposed a set of boundary conditions at $X = X_{cr}$ which ensure a finite spacetime curvature and also stop Hawking radiation. The RST boundary conditions have been subsequently criticized [9], because they are incompatible with natural quantum

mechanical boundary conditions for the matter fields. As an alternative, it has been proposed that boundary conditions should be imposed on all fields along a timelike curve, the analog of the origin of radial coordinates in four dimensional gravity [9].

Here we will be concerned with a completely different solution to the unending Hawking radiation. As proposed by Banks and O'Loughlin (BO) [10], if the two dimensional theory is modified in order to admit black holes of Reissner Nordstrom type, then, in the extremal limit, the temperature vanishes and a stable end point is achieved. The extremal black holes proposed by BO [10] are plausible remnants but, unfortunately, the models are difficult to handle and they have not been analytically solved at the semiclassical level (see also [11]).

Following BO's idea, we will modify the CGHS action by adding a nonperturbative correction to the potential term which allows classically an extremal black hole solution. We will show that it is possible to construct the conformal field theory associated to this model and thus perform the quantization procedure àlla David, Distler and Kawai [12]. Even though this conformal field theory is not easily analytically solvable, it is possible to show that the extremal geometry is not a solution of the full quantum system. Thus, we will conclude that this quantization procedure spoils the hope to stop Hawking radiation by this means.

In Section 2 we perform the quantization of an arbitrary dilaton gravity lagrangian by generalizing the method introduced by de Alwis (dA) [3]. In Section 3, the unending evaporation problem of two dimensional black holes is discussed and one particular solution is analyzed both classically and quantum mechanically. Conclusions are presented in Section 4.

2. The most general renormalizable dilaton gravity lagrangian can be written as

$$\mathcal{L} = \sqrt{-g}\left[D(\phi)R + G(\phi)(\bigtriangledown\phi)^2 + V(\phi)\right] \tag{1}$$

where $D(\phi)$, $G(\phi)$ and $V(\phi)$ are arbitrary functions which, in order to recover the CGHS theory in the weak coupling limit, should satisfy

$$D(\phi) \to \frac{G(\phi)}{4} \to e^{-2\phi} \quad , \qquad V(\phi) \to 4\lambda^2 e^{-2\phi} \tag{2}$$

as $e^{2\phi} << 1$.

As shown by BO, for computational reasons, it is convenient to eliminate the G term. This can be done in a nonsingular way when $D' = \frac{dD}{d\phi} \neq 0$, by performing a conformal transformation $g_{\mu\nu} = e^{2S(\phi)}\hat{g}_{\mu\nu}$ with $2S'(\phi)D'(\phi) = -G(\phi)$. Thus the lagrangian reduces to

$$\mathcal{L} = \sqrt{-\hat{g}}\left[D(\phi)\hat{R} + W(\phi)\right] \tag{3}$$

where $W = Ve^{2S}$. In order to quantize the system we generalize to this case the procedure originally introduced by dA [3]. In conformal gauge $\hat{g} = e^{2\hat{\rho}}\tilde{g}$, the action (3), supplemented with the Liouville action, takes the semiclassical form

$$S[\tilde{g}] = \frac{1}{4\pi}\int d^2\sigma\sqrt{-\tilde{g}}\left\{[D(\phi) - \kappa\hat{\rho}]\tilde{R} + 2D'(\phi)\tilde{\bigtriangledown}\hat{\rho}\tilde{\bigtriangledown}\phi - \kappa(\tilde{\bigtriangledown}\hat{\rho})^2 + W(\phi)e^{2\hat{\rho}}\right\} \tag{4}$$

which can be written as a non linear σ-model

$$I[X,\tilde{g}] = -\frac{1}{4\pi}\int d^2\sigma[\frac{1}{2}\tilde{g}^{ab}G_{\mu\nu}\partial_a X^\mu\partial_b X^\nu + \tilde{R}\Phi(X) + T(X)] \tag{5}$$

where $X \equiv (\phi, \hat{\rho})$, $G_{\mu\nu}$ is a metric in the space of fields $(\phi, \hat{\rho})$ and Φ and T are the dilaton and tachyon fields respectively. In order to define a conformal field theory, the couplings G, Φ and T must solve the β-function equations with the weak coupling boundary conditions

$$\begin{gathered} G_{\phi\phi} = 0 \qquad G_{\phi\hat{\rho}} = -2D'(\phi) \qquad G_{\hat{\rho}\hat{\rho}} = 2\kappa \\ \Phi = -D(\phi) + \kappa\hat{\rho} \qquad T = -W(\phi)e^{2\hat{\rho}} \end{gathered} \tag{6}$$

Recall that the effective coupling is $|D'(\phi)|^{-1}$ and thus, the weak coupling region is characterized by $|D'|^{-1} << 1$.

Parametrizing the target space metric as

$$ds^2 = h(\phi)d\phi^2 - 4D'[1 + \bar{h}(\phi)]d\hat{\rho}d\phi + 2\kappa[1 + \bar{\bar{h}}(\phi)]d\hat{\rho}^2 \tag{7}$$

with $h(\phi) = O(D'^0)$ and $\bar{h}(\phi) = O(D'^{-1})$, it can be easily shown that $\mathbf{R}(G_{\mu\nu}) = 0$ if $\bar{\bar{h}} = 0$. Therefore in this case it is possible to find a Minkowskian coordinate system. Defining

$$
\begin{aligned}
X &\equiv \frac{2}{\kappa}\int d\phi \left[\frac{D'^2}{4}(1+\bar{h})^2 - \frac{1}{8}\kappa h\right]^{1/2} \\
Y &\equiv \left[\hat{\rho} - \frac{1}{\kappa}\int d\phi (1+\bar{h}(\phi))D'(\phi)\right]
\end{aligned}
\tag{8}
$$

the target space interval reads $ds^2 = 2\kappa(-dX^2 + dY^2)$ and the action (5) can be written as

$$
I = \frac{1}{4\pi}\int d^2\sigma \left[-4\kappa\partial_+ X\partial_- X + 4\kappa\partial_+ Y\partial_- Y - T(X,Y)\right] \tag{9}
$$

The gravitational β-function

$$
\beta_{\mu\nu} = \mathbf{R}_{\mu\nu} + 2\nabla^G_\mu \partial_\nu \Phi - \partial_\mu T\partial_\nu T + ... = 0 \tag{10}
$$

is easily seen to be solved, neglecting terms of $O(T^2)$ and imposing the boundary conditions (6), by $\Phi = \kappa Y$. Replacing this linear dilaton solution in the dilaton β-function

$$
\beta_\Phi = -\mathbf{R} + 4G^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - 4\nabla^2_G\Phi + \frac{N-24}{3} + G^{\mu\nu}\partial_\mu T\partial_\nu T - 2T^2 + ... = 0 \tag{11}
$$

κ turns out to be $\kappa = \frac{24-N}{6}$, N being the number of conformal fields.

T(X,Y) should satisfy the tachyon β-function, namely

$$
\beta_T = -2\nabla^2_G T + 4G^{\mu\nu}\partial_\mu\Phi\partial_\nu T - 4T + \ldots = 0 \tag{12}
$$

If we restrict to functions $W(\phi) = const = 4\lambda^2$, a possible solution of (12) to leading order, is $T(X,Y) = -4\lambda^2 e^{\beta X + \alpha Y}$ with $\kappa^{-1}(\beta^2 - \alpha^2) + 2\alpha - 4 = 0$. In order to match the weak coupling boundary conditions we must have $\alpha = 2$, $\beta = \mp 2$, where the upper (lower) sign corresponds to $D' < 0 (D' > 0)$.

We have quantized the 2D models using the reduced lagrangian (3) instead of the original lagrangian (1). In performing the quantization with (1), the target space metric would be parametrized as

$$
ds^2 = -2G(\phi)(1+l)d\phi^2 - 4D'(\phi)(1+\bar{l})d\phi d\rho + 2\kappa d\rho^2 \tag{13}
$$

A comparison of Eqns. (7) (with $\bar{\bar{h}} = 0$) and (13), shows that both quantization procedures are equivalent if

$$
\begin{aligned}
h &= -2G(\phi)(1+l) - 4D'(\phi)(1+\bar{l})S' + 2\kappa[S'(\phi)]^2 \\
\bar{h} &= \bar{l} - \kappa\frac{S'(\phi)}{D'(\phi)}
\end{aligned}
\tag{14}
$$

We will use these relations in the next Section.

3. Let us now consider the CGHS model with an additional tachyon contribution. Namely, let us take in (9) for $\kappa > 0$

$$
T(X,Y) = -4\lambda^2 e^{2(Y-X)} + \mu e^{2(X+Y)} \equiv -e^{2\hat{\rho}}W(\phi) \tag{15}
$$

which is also a solution of the tachyon β-function (12) satisfying the weak coupling boundary conditions (6). Indeed, as $\phi \to -\infty$, it is easily seen that

$$
T(X,Y) = -e^{2\hat{\rho}}\left[4\lambda^2 - \mu e^{-\frac{4}{\kappa}e^{-2\phi}}\right] \to -4\lambda^2 e^{2\hat{\rho}} \tag{16}
$$

(note that this analysis does not hold for $\kappa < 0$). The additive term may be viewed as a nonperturbative quantum correction to $W(\phi)$ which vanishes very rapidly in the weak coupling region, but introduces interesting consequences for the black hole solutions. Indeed, as shown by BO, models in which $W(\phi)$ has one zero have extremal black hole solutions, and are thus plausible remnants to solve Hawking's evaporation paradox. Similar nonperturbative corrections have been proposed in Ref. [13].

Let us first consider the reduced lagrangian

$$
\mathcal{L} = \sqrt{-\hat{g}}\,[D(\phi)R + W(\phi)] \tag{17}
$$

where $W(\phi) = 4\lambda^2 - \mu e^{-\frac{4}{\kappa}e^{-2\phi}}$ and $D(\phi) = e^{-2\phi}$. In terms of the X, Y fields defined by Eq.(8) with $h = \bar{h} = 0$,

$$
X = -\frac{1}{\kappa}e^{-2\phi} \quad , \qquad Y = \rho - \phi - \frac{1}{\kappa}e^{-2\phi} \quad , \tag{18}
$$

this lagrangian reads

$$
\mathcal{L} = -8\kappa\partial_+ X\partial_- X + 8\kappa\partial_+ Y\partial_- X + 4\lambda^2 e^{2(Y-X)} - \mu e^{2(X+Y)} \tag{19}
$$

The equations of motion are given by

$$\partial_{+-}(Y-3X) = -\frac{2\lambda^2}{\kappa}e^{2(Y-X)} \quad , \qquad \partial_{+-}(Y-X) = -\frac{\mu}{2\kappa}e^{2(Y+X)} \tag{20}$$

We would like to see if these equations admit an extremal black hole solution. In the neighborhood of an extremal horizon, $ds^2 = -(\alpha r - 1)^2 d\tau^2 + \frac{dr^2}{(\alpha r-1)^2} = e^{2\rho(\sigma)}[-d\tau^2 + d\sigma^2]$, where $\rho(\sigma) \to -ln\alpha|\sigma|$ as $\sigma \to -\infty$. Assuming that near the horizon $\phi = \phi_0 + g(\sigma)$, with $g(\sigma) \to 0$, the above equations lead to

$$\frac{1}{\sigma^2} = -\frac{8\lambda^2}{\alpha^2\sigma^2}X(\phi_0) + O(\frac{1}{\sigma^3}) \quad , \qquad \frac{1}{\sigma^2} = -\frac{2\mu}{\alpha^2\sigma^2}X(\phi_0)e^{4X(\phi_0)} + O(\frac{1}{\sigma^3}) \tag{21}$$

It is easy to see that, to lowest order in σ^{-2}, $\alpha^2 = -8\lambda^2 X(\phi_0)$, $\quad X(\phi_0) = -\frac{1}{4}ln(\frac{\mu}{4\lambda^2})$ is a solution (notice that, as $X < 0$, the solution exists only if $\frac{\mu}{4\lambda^2} > 1$). Therefore, the equations admit an extremal horizon.

Since this was an approximate analysis, we would like to confirm the validity of the result. Eqns (20) imply that $\frac{dX}{d\sigma} = Ce^{2(Y-X)}$, where C is an integration constant. As a consequence

$$e^{2(Y-X)} = -\frac{1}{\kappa C^2}\int dX \;\; W(X) \equiv -\frac{1}{\kappa C^2}P(X) \tag{22}$$

A horizon is a point where $e^{2\rho} = -\frac{1}{\kappa X}e^{2(Y-X)} = 0$, ρ being the Liouville field of the CGHS metric related to the new metric $\hat{g}_{\mu\nu}$ by $\rho = \hat{\rho} + \phi$. Therefore, the zeros of $P(X)$ define the horizons. When P has a linear zero the behaviour of the solutions is exactly as in the CGHS model. When W has one zero, the function P will have two. If there exists a particular value of the ADM mass for which both zeros coincide, it would define an extremal black hole [10]. Quantum matter fields in this background would not Hawking radiate.

The function $P(X)$ defined in Eqn.(22) is given by

$$P(X) = 4\lambda^2 X - \frac{\mu}{4}e^{4X} + \frac{A}{\kappa} \tag{23}$$

where A is an integration constant, related to the ADM mass of the black hole by $M = \frac{A}{4\lambda}$.

$P(X)$ has a maximum at $X(\phi_0)$. A simple analysis shows that it has one simple zero when $A > \frac{\mu\kappa}{4}$, two simple zeros for $\frac{\mu\kappa}{4} > A > A_{crit} = 4\lambda^2\kappa(\frac{1}{4} - X(\phi_0))$, a double zero for

$A = A_{crit}$ and no zeros for $A < A_{crit}$. Therefore, the model we have discussed possesses a classical extremal solution with mass $M_{crit} = \frac{A_{crit}}{4\lambda}$. The value of X at the horizon is $X(\phi_0)$, as anticipated from the approximate calculation.

It is well known that the Reissner-Nordstrom extremal geometry leads to null Hawking radiation. Applying the procedure introduced by Christensen and Fulling [14] to compute the Hawking temperature from the trace of the energy momentum tensor, we find that for this model, T is proportional to $X\frac{d}{dX}\left(\frac{P(X)}{X}\right)|_{X_h}$, where X_h is the value of X at the horizon nearest to the weak coupling region. Therefore there are two qualitatively different possibilities for the Hawking temperature, depending on the multiplicity of the roots of P. If P has two simple roots, i.e. two non coincident horizons, then the temperature will have a non zero finite value. If, on the contrary, both horizons coincide (extremal geometry), then P will have a double zero and the temperature will vanish.

It is interesting to note that this behaviour holds for any model in which $W(\phi)$ has one zero. Unfortunately, as we shall now show, the extremal geometry is not a solution of the full quantum system.

In order to do this we must solve the equations of motion following from (9), using (15). Namely,

$$\partial_{+-}(X+Y) = \frac{2\lambda^2}{\kappa}e^{2(Y-X)} \quad , \qquad \partial_{+-}(X-Y) = \frac{\mu}{2\kappa}e^{2(Y+X)} \tag{24}$$

where

$$X = \frac{2}{\kappa}\int d\phi\sqrt{e^{-4\phi}(1+\bar{h})^2 - \frac{1}{8}\kappa h} \quad , \qquad Y = \left(\hat{\rho} - \frac{e^{-2\phi}}{\kappa} + \frac{2}{\kappa}\int d\phi \bar{h} e^{-2\phi}\right) \tag{25}$$

and $\hat{\rho} = \rho - \phi$.

Even though the general analytical solution of (24) is not easy to find, we will proceed with the approximate analysis performed in the classical case, Eqns. (20). In order to do this, we choose Strominger's $l = -e^{-2\phi}$ and $\bar{l} = -2e^{-2\phi}$ so as to ensure no ghost radiation [4]. These give, according to Eqn.(14) $h = 2\kappa - 8$ and $\bar{h} = e^{2\phi}(\frac{\kappa}{2} - 2)$. Replacing in (25), X and Y turn out to be

$$X = \frac{2}{\kappa}\int d\phi\sqrt{e^{-4\phi}-(4-\kappa)e^{-2\phi}+(4-\kappa)}\ \ , \qquad Y = \rho - \frac{1}{\kappa}\left(e^{-2\phi}+4\phi\right) \tag{26}$$

As ρ and ϕ vary between $+\infty$ and $-\infty$, so does Y. X also shares this property for positive κ (note that $0<\kappa<4$). As a consequence, equation (9) defines a conformal field theory without boundaries and thus, unless boundary conditions are imposed by hand at a timelike curve [9], the only way to stop Hawking radiation is through an extremal black hole geometry. Let us see if the equations (24) admit such possibility.

Let us consider first a given finite value of the coupling, $\phi\to\phi_0$, since this is where the extremal classical solution was found. Thus,

$$X \simeq F(\phi_0)\ \ , \qquad Y\simeq -ln\alpha|\sigma| + G(\phi_0) \tag{27}$$

where F and G can be read from Eq.(26). Eqns. (24) turn out to be incompatible upon replacement of these expressions. Indeed, they yield

$$\frac{1}{\sigma^2} = -\frac{8\lambda^2}{\kappa}\frac{e^{2[G(\phi_0)-F(\phi_0)]}}{\alpha^2\sigma^2} + O(\frac{1}{\sigma^3})\ \ , \qquad \frac{1}{\sigma^2} = \frac{2\mu}{\kappa}\frac{e^{2[G(\phi_0)+F(\phi_0)]}}{\alpha^2\sigma^2} + O(\frac{1}{\sigma^3}) \tag{28}$$

and both rhs have opposite signs, contrary to the classical case. The difference can be traced back to the conformal anomaly. Indeed, the difference between the Lagrangians (19) and (9) is the conformal anomaly term $4\kappa\partial_+(Y-X)\partial_-(Y-X)$. This term flips the relative sign of the field Y in the equations. Therefore the classical extremal horizon at finite coupling is erased by quantum effects. Notice that this result is general, i.e. independent of the particular quantum correction functions h and $\bar{h}$. However this does not mean that the extremal geometry is not a solution, since the double horizon may still be allowed in the weak or strong coupling regions, to which we now turn.

Consider weak coupling. As $\phi\to-\infty$,

$$X \simeq \frac{1}{\kappa}\left[-e^{-2\phi}+(\kappa-4)\phi\right]\ \ , \qquad Y=\rho-\frac{1}{\kappa}e^{-2\phi}-\frac{4}{\kappa}\phi \tag{29}$$

Replacing in (24), it is easy to see that $\rho-\phi$ is a free field and the CGHS solution is obtained, as expected. There is no extremal horizon at weak coupling.

The remaining possibility is $\phi \to +\infty$, i.e. strong coupling. In this case,

$$
\begin{aligned}
X &\simeq \frac{2}{\kappa}\frac{\sqrt{4-\kappa}}{2}\phi = af(\sigma), \\
Y &\simeq \left(\rho - \frac{4}{\kappa}\phi\right) = -ln\alpha|\sigma| - f(\sigma)
\end{aligned}
\tag{30}
$$

where $f(\sigma) \to +\infty$ when $\sigma \to -\infty$ and $0 < a = \sqrt{\frac{4-\kappa}{4}} < 1$. Replacing in (24) the equations read

$$
\begin{aligned}
\frac{1}{\sigma^2} - (1-a)f'' &= -\frac{8\lambda^2}{\kappa}\frac{e^{-2(1+a)f}}{\alpha^2\sigma^2} \\
\frac{1}{\sigma^2} - (1+a)f'' &= \frac{2\mu}{\kappa}\frac{e^{-2(1-a)f}}{\alpha^2\sigma^2}
\end{aligned}
\tag{31}
$$

Thus, in the limit $\sigma \to -\infty$ the exponentials $e^{-2(1\pm a)f}$ tend to zero and both equations are incompatible unless $a = 0$, i.e. the uninteresting case $N = 0$, where there is no Hawking radiation.

Since the analysis has covered all possible coupling regions, namely $-\infty < \phi < \infty$, the conclusion is then that the equations (24) do not admit an extremal black hole solution. It is important to stress that this result depends strongly on the particular form of the additional tachyon contribution. Had we considered a different potential, such as $W(\phi) = 4\lambda^2 - \mu e^{-\frac{e^{-2\phi}}{b}}$ with $b \gg \kappa$, the extremal black hole would have survived. However, $b = \frac{\kappa}{4}$ is fixed by the requierement of conformal invariance (vanishing of the tachyon β-function).

4. At the classical level, the theory defined by Eq.(17) admits black holes of the Reissner Nordstrom type. One would be tempted to argue that, in this model, the black hole evaporation would take place until the extremal limit is reached. In order to discuss this issue we should consider the dynamical problem. However, we have shown in the previous Section that the extremal geometry is destroyed by quantum effects.

The example above calls for discreet prudence before extracting quick conclusions from classical results. In particular, the belief related to the possibility that extremal black hole remnants solve Hawking's evaporation paradox might not be correct. We have considered one particular example where the non-perturbative term in the potential was chosen to

define a conformally invariant theory. The quantum equations of motion are qualitatively different from the classical ones. Thus, a small modification of the classical geometry, as considered in [11], is not a solution. The extremal horizon disappears.

Our results do not rule out extremal remnants as the end point of black hole evaporation. They strongly suggest that, in these models, it is not always safe to appeal to the benefit of quantum matter corrections, before including quantum gravitational and dilaton effects.

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